AP Calculus AB

Practice Exam

Calculus AB
Section 1, Part A
Time - 55 minutes
Number of questions - 28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM

Directions: Solve all of the problems that follow. Available space on the page may be used for scratch work.

In this exam:

(1) The domain of a function in this exam is assumed to be all real numbers for which the function is defined, unless specified otherwise.

(2) The inverse notation f^{-1} or the prefix "arc" may be used to indicate an inverse function. For example, the inverse tangent of x may be written as $\arcsin(x)$ or as $\tan^{-1}(x)$.

1. Using the substitution $u = 3x - 2$, the integral $\int_0^3 \sin(3x-2)\,dx$ is equivalent to

A) $\int_{-2}^{7} \sin u \, du$

B) $\int_{0}^{3} \sin u \, du$

C) $\frac{1}{3}\int_{-2}^{7} \sin u \, du$

D) $\frac{1}{3}\int_{0}^{3} \sin u \, du$

E) $\frac{1}{3}\int_{0}^{7} \sin u \, du$

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2. $y = (x^4 + 2)^2$. Find $\frac{dy}{dx}$.

A) $8x^3(x^4 + 2)$

B) $(x^4 + 2) \cdot 4x^3$

C) $2(4x^3 + 2)$

D) $2(x^4 + 2)$

E) $(4x^3)^2$

3. A line through the point (2, -2) is tangent to function f at the point (-2, 6). What is $f'(-2)$?

A) -8 B) -2 C) $-\frac{1}{2}$ D) 6 E) Undefined

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4. Grain is pouring out of an opening in the bottom of a grain silo. The rate at which the height, *h*, of the grain in the silo is changing with respect to time, *t*, is proportional to the square root of the height. Which of the following is a differential equation describing this situation?

A) $\frac{dh}{dt} = k\sqrt{t}$

B) $\frac{dh}{dt} = \frac{k}{\sqrt{h}}$

C) $\frac{dh}{dt} = k\sqrt{h}$

D) $h(t) = k\sqrt{t}$

E) $h(t) = k\sqrt{h}$

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5. Where is the graph of $f(x) = xe^{2x}$ concave up?

A) $x > -1$ B) $x < -1$ C) $x > \frac{1}{2}$ D) $x < -\frac{1}{2}$ E) $x > 0$

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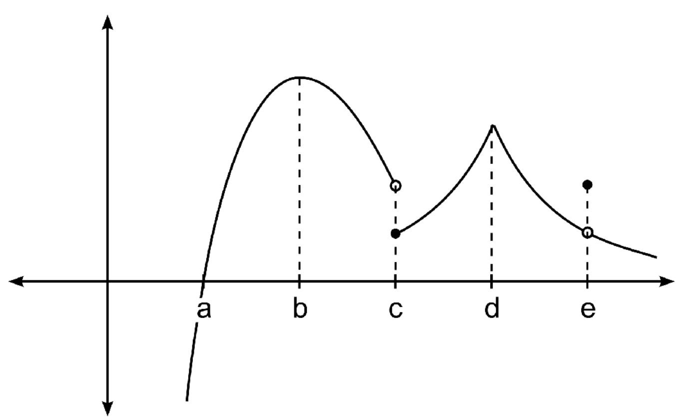
6. Shown is a graph of function f.



At which value of x is f continuous but not differentiable?

A) a B) b C) c D) d E) e

7. The table shows values of f', the derivative of function f. Although f' is continuous over all real numbers, only selected values of f' are shown.

x	-2	-1	0	1	2	3	4	5	6
$f'(x)$	4.1	2.3	1.2	0	-0.6	-0.8	-2	0	1.5

If f' has exactly two real zeros, then f is increasing over which of the following intervals?

A) $1 < x < 5$

B) $x < 1$ or $x > 5$

C) $x > 5$ only

D) $x > 1$

E) $x > 3$

---

8. $y = x^3 \sin(3x)$. $\dfrac{dy}{dx} =$

A) $3x^2 \cos(3x)$

B) $9x^2 \cos(3x)$

C) $3x^2(\sin(3x) + \cos(3x))$

D) $3x^2(\sin(3x) - x\cos(3x))$

E) $3x^2(\sin(3x) + x\cos(3x))$

9. At each point (x, y) on a graph of function f, the graph has a slope equal to $4x - 4$. If the graph of f goes through the point (2, 3), then $f(x) =$

A) $2x^2 - 4x - 4$

B) $2x^2 - 4x$

C) $2x^2 - 1$

D) $2x^2 - 4x + 3$

E) $16x - 12$

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10. The derivative of function g is given by $g'(x) = \dfrac{4}{x^2} - x$. When is g increasing?

A) $\left(-\infty, \sqrt[3]{4}\right)$

B) $\left(\sqrt[3]{4}, \infty\right)$

C) $(-1, 0)$

D) $(-\infty, 1)$

E) $(1, \infty)$

11. Function f is defined as $f(x) = \begin{cases} -x+5, & x \le 2 \\ -\frac{1}{2}x+4, & x > 2 \end{cases}$

Which of the following statements is true:

I. f is continuous at $x = 2$.
II. $\lim_{x \to 2} f(x)$ exists.
III. f is differentiable at $x = 2$

A) None of the statements are true.
B) I only
C) II only
D) I and II only
E) I, II, and III

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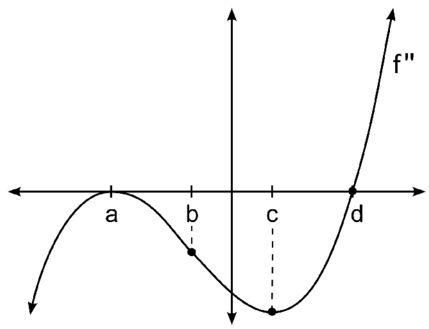
12. $\int_0^2 e^{-3x}dx =$

A) $-3e^{-6}$ B) $e^{-6} - 1$ C) $\frac{1}{3} - \frac{e^{-6}}{3}$ D) $3 - 3e^{-6}$ E) $-\frac{e^{-6}}{3}$

13. Shown is a graph of f'', the second derivative of function f. The curve is given by the equation $f'' = (x-a)^2(x-d)$. The graph of f has inflection points at which values of x ?



A) b only
B) c only
C) a and d
D) a and c
E) d only

---

14. Function f is defined such that for all $x \geq 0$, the line $y = 3$ is a horizontal asymptote. Which of the following must be true?

A) $f(3)$ is undefined.

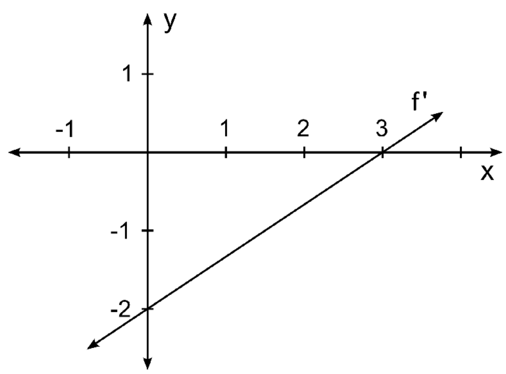
B) $f(x) \neq 3$ for all $x \geq 0$.

C) $\lim_{x \to \infty} f(x) = 3$

D) All of the above.

E) None of the above.

15. Shown in the diagram is a graph of f', the derivative of function f. If $f(3) = 3$, then $f(1) = $?



A) $\frac{10}{3}$ B) $\frac{13}{3}$ C) $-\frac{5}{3}$ D) 4 E) 6

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16. $y = \frac{3x + 4}{4x + 3}$ $\frac{dy}{dx} =$

A) $\frac{7}{(4x+3)^2}$ B) $\frac{24x-25}{(4x+3)^2}$ C) $\frac{-7}{(4x+3)^2}$ D) $\frac{24x+25}{(4x+3)^2}$ E) $\frac{3}{4}$

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17. $\frac{d}{dx}\int_0^{x^3} \cos\left(t^2\right) dt$ =

A. $-\sin\left(x^2\right)$

B. $\sin\left(x^3\right)$

C. $\sin\left(x^6\right)$

D. $3x^2 \cos\left(x^6\right)$

E. $3x^2 \cos\left(x^3\right)$

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18. $\int_0^{\pi/6} \sin x \, dx$ =

A) $-\frac{\sqrt{3}}{2} + 1$

B) $-\frac{\sqrt{3}}{2}$

C) $\frac{\sqrt{3}}{2} - 1$

D) $\frac{\sqrt{2}}{2}$

E) $-\frac{\sqrt{3}}{2} - 1$

19. Let f be the function defined by $f(x) = x^3 - 3x + 5$. The equation of the line tangent to the graph of f at $x = 2$ is

A. $y = 9x - 18$

B. $y = 3x + 1$

C. $y = 9x - 11$

D. $y = 3x - 6$

E. $y = 9x + 25$

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20. $\lim\limits_{x \to \infty} \dfrac{2x^3 + 3x^2 - 4x + 2}{4x^3 - x^2 + 5x - 3}$

A) $-\dfrac{1}{2}$ B) 0 C) $\dfrac{1}{2}$ D) 1 E) 2

21. A particle's position at any time is given by the equation $x(t) = t^3 - 12t^2 + 36t - 15$. At what time is the particle at rest?

A) $t = 3$ and $t = 6$

B) $t = 2$ and $t = 6$

C) $t = 2$ and $t = 4$

D) $t = 2$ only

E) $t = 4$ only

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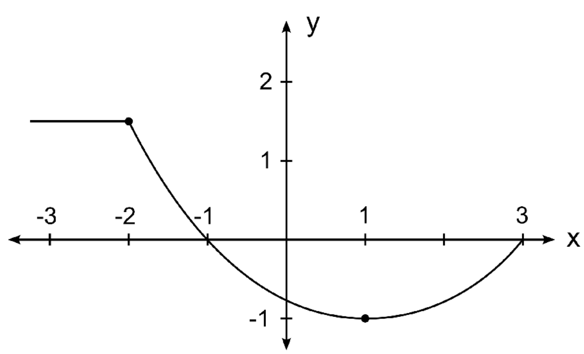
22. Shown is a graph of f', the derivative of function f.



Which of the following statements is true?

A) f is not differentiable at $x = -2$.
B) f has a local minimum at $x = -1$.
C) f is increasing from $x = 1$ to $x = 3$.
D) f is increasing from $x = -3$ to $x = -1$.
E) f is decreasing from $x = -2$ to $x = 1$.

23. What is the slope the line tangent to the curve $2x^2y - 3 = y^2 + 3x^2$ at the point (2, 3) ?

A) 9 B) $-\frac{3}{4}$ C) $-\frac{15}{2}$ D) $-\frac{3}{7}$ E) −6

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24. $\int x^3 \cos\left(x^4\right) dx =$

A) $-\frac{x^4}{4}\sin\left(x^4\right) + C$

B) $\frac{x^4}{4}\sin\left(x^4\right) + C$

C) $-\frac{1}{4}\sin\left(x^4\right) + C$

D) $\frac{1}{4}\sin\left(x^4\right) + C$

E) $\frac{x^4}{4}\sin\left(\frac{x^4}{4}\right) + C$

25. Function f is defined by the equation $f(x) = x^3 - 2x$. If $g(x) = f^{-1}(x)$ and $g(4) = 2$, what is $g'(4)$?

A) $\frac{1}{10}$ B) $\frac{1}{46}$ C) 46 D) 10 E) $\frac{23}{5}$

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26. $f(x) = \ln\left(e^{-4x} + 5x + 4\right)$. Find $f'(0)$.

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $-\frac{1}{5}$ D) $-\frac{1}{4}$ E) does not exist.

27. Function f is a twice differentiable function with $f'(x) > 0$ and $f''(x) < 0$ for all real numbers x. If $f(2) = 5$ and $f(3) = 9$, what is a possible value for $f(4)$?

A) 7 B) 9 C) 12 D) 13 E) 17

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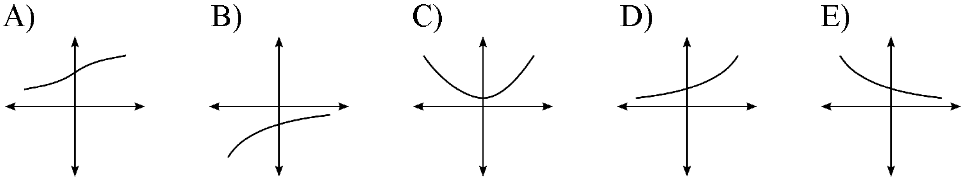
28. $f(x)$, $f'(x)$, and $f''(x)$ are all positive for any real number x. Which of the following graphs could be a graph of f?

A) B) C) D) E)



End of Section 1, Part A

If you finish before the time limit for this part, check your work on this part only.

Do move on to the next part until you are told to by the test administrator.

Calculus AB
Section 1, Part B
Time - 50 minutes
Number of questions - 17

A GRAPHING CALCULATOR MAY BE REQUIRED TO SOLVE SOME QUESTIONS ON THIS PART OF THE EXAM

Directions: Solve all of the problems that follow. Available space on the page may be used for scratch work. You may not return to the previous section of the exam.

In this exam:

(1) The domain of a function in this exam is assumed to be all real numbers for which the function is defined, unless specified otherwise.

(2) The inverse notation f^{-1} or the prefix "arc" may be used to indicate an inverse function. For example, the inverse tangent of x may be written as arcsin(x) or as $\tan^{-1}(x)$.

(3) The exact numerical answer for a problem may not be listed as one of the given choices. When this is the case, choose the value that is the closest approximation to exact value.

76. The derivative of function f is given by $f' = \dfrac{2x - x^2}{e^{2x}}$. At what values of x does the graph of f have an inflection point?

A. 0

B. 0.382

C. 0.775

D. 1.136

E. The graph has no inflection point.

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77. The region in the x-y plane bounded by the lines $x = 1$, $x = 3$, $y = 2$, and $y = \ln x$ is the base of a solid. Each cross section of the solid perpendicular to the x-axis is a square. The volume of the solid is

A. 3.85 B. 2.70 C. 2.97 D. 6.70 E. 5.41

78. Function f is differentiable and $f(3) = 6$ and $f'(3) = 4$. Function g is defined as $g(x) = \dfrac{f(x)}{x}$. What is the equation of the line tangent to function g at $x = 3$.

A. $y - 2 = 4(x - 3)$

B. $y - 2 = \frac{4}{3}(x - 3)$

C. $y - 6 = 4(x - 3)$

D. $y - 2 = \frac{2}{3}(x - 3)$

E. $y - 3 = \frac{2}{3}(x - 2)$

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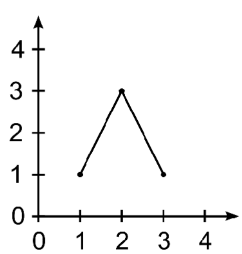
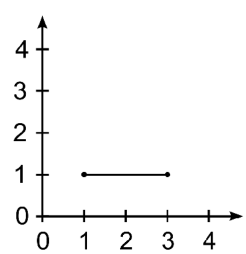
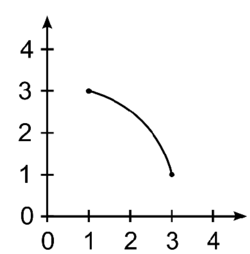
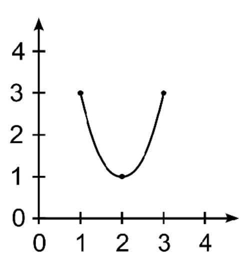
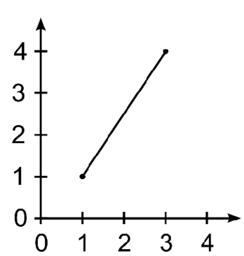
79. Which could be a graph of f such that $\dfrac{1}{3-1}\int_1^3 f(x)\,dx = 2$?

A.


B.


C.


D.


E.


80. The acceleration of a particle moving along the x-axis is given by $a(t) = \ln\left(3^t + 2\right)$. At time $t = 2$, the velocity of the particle is 2. What is the velocity at time $t = 3$?

A. -0.441

B. 3.099

C. 3.178

D. 4.872

E. 5.313

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81. Shown are selected values for functions *f*, *g*, *h*, *j*, and *k*, all of which are twice differentiable in the closed interval [1, 4]. Which of the functions has a negative first derivative and a positive second derivative?

A.

x	f (x)
1	14
2	12
3	10
4	8

B.

x	g (x)
1	14
2	13
3	11
4	8

C.

x	h (x)
1	14
2	11
3	9
4	8

D.

x	j (x)
1	8
2	9
3	11
4	14

E.

x	k (x)
1	9
2	11
3	13
4	14

82. A particle is moving such that its velocity at any time t is given by $v(t) = 2 + 2.5\sin(0.6t)$. What is the acceleration of the particle when $t = 3$?

A. -0.341

B. -0.568

C. -0.947

D. 1.24

E. 1.46

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83. Function f is defined as $f(x) = \int_0^x \cos\left(\frac{t^2}{2}\right)$ for $0 \le x \le 3.8$. On what interval is f decreasing?

A. $0 \le x \le 1.772$

B. $0 \le x \le 2.506$

C. $1.772 \le x \le 3.07$

D. $0 \le x \le 3.07$

E. $3.545 \le x \le 3.8$

84. If a circle's radius is increasing at a constant rate of $0.3\ \frac{\text{m}}{\text{s}}$, what is the rate of change of the area of the circle when the circle's circumference is 6π meters?

A. $0.09\pi\ \frac{\text{m}^2}{\text{s}}$

B. $0.6\pi\ \frac{\text{m}^2}{\text{s}}$

C. $1.8\pi\ \frac{\text{m}^2}{\text{s}}$

D. $6\pi\ \frac{\text{m}^2}{\text{s}}$

E. $9\pi\ \frac{\text{m}^2}{\text{s}}$

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85. Function f is continuous on the closed interval [-1, 3] and differentiable on the open interval (-1, 3). If $f(-1) = 1$ and $f(3) = 3$, which of the following statements must be true?

A. There exists a number c on (-1, 3) such that $f(c) = \frac{1}{2}$.

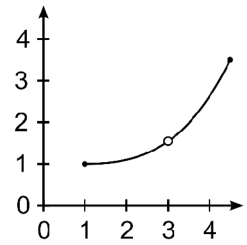
B. There exists a number c on (-1, 3) such that $f'(c) = 0$

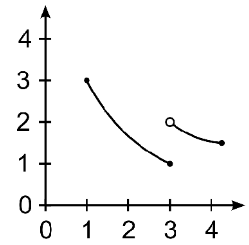
C. There exists a number c on (-1, 3) such that $f(c) = 0$

D. There exists a number c on (-1, 3) such that $f(c) \geq f(x)$ for all x

E. There exists a number c on (-1, 3) such that $f'(c) = \frac{1}{2}$

86. For which of the graphs shown does the $\lim_{x \to 3} f(x)$ exist?

I.


II.


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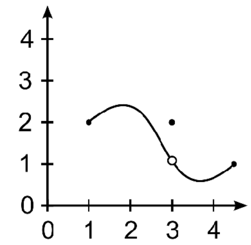

A) I only
B) II only
C) I and II only
D) III only
E) I and III only

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87. $f'(x) = \cos(2x)$. How many relative extrema does function $f(x)$ have on the interval $\frac{\pi}{2} < x < 2\pi$.

A. 2 B. 3 C. 4 D. 5 E. 7

88. A remote control helicopter is launched. During the first 6 seconds of flight, the rate of change of the helicopter's altitude is given by the equation $r(t) = -t^3 + 5t^2 - 8$. Which of the following expressions represents the helicopter's change in altitude during the time that the altitude is increasing?

A. $\int_0^{10.518} r(t)\,dt$

B. $\int_0^{10.518} r'(t)\,dt$

C. $\int_{1.515}^{4.626} r(t)\,dt$

D. $\int_{1.515}^{4.626} r'(t)\,dt$

E. $\int_0^{6} r(t)\,dt$

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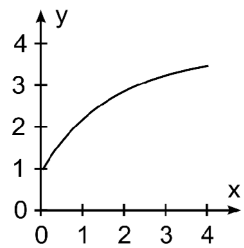
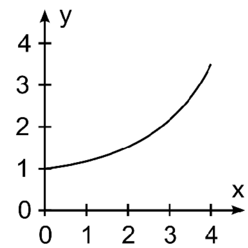
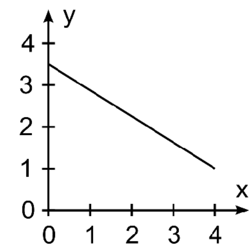
89. A potato is baked at a temp of 400°F. At time $t = 0$ it is taken out of the oven and allowed to cool in a 72°F room. The rate of change of the potato's temperature is given by the equation $r(t) = -82e^{-0.25t}$ where t is the time in minutes. What is the potato's temperature, to the nearest degree, after it has cooled for four minutes?

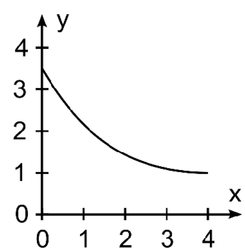
A. 104 B. 193 C. 207 D. 333 E. 370

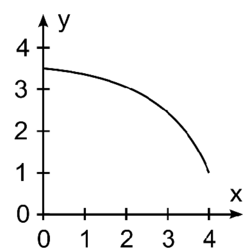
90. The integral $\int_0^5 g(x)\,dx$ is under approximated by a trapezoidal sum and over approximated by a left Riemann sum. Which of the following could be a graph of $y = g(x)$?

A.


B.


C.


D.


E.


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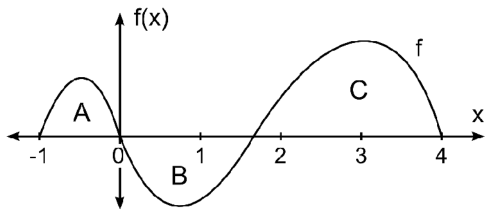
91. An object is moving along the x-axis such that its velocity in m/s is given by $v(t) = 2e^{x} + x\sin(x)$. What is the average velocity of the object from time $t = 0$ to time $t = 4$?

A. 25.95 B. 26.04 C. 27.26 D. 54.08 E. 104.17

92. A graph of function $f(x)$ on the interval [-1, 4] is shown. Regions A, B, and C have areas of 1, 2, and 3 respectively. What is $\int_{-1}^{4} \left(f(x) + 2 \right) dx$?



A. 2 B. 4 C. 10 D. 12 E. 16

End of Section 1, Part B

If you finish before the time limit for this part, check your work on this part only.

Do move on to the next part until you are told to by the test administrator.

Calculus AB
Section 2, Part A
Time - 30 minutes
Number of problems - 2

A graphing calculator is required for these problems

1. A hot air balloon is launched at time $t = 0$. Its altitude in meters is modeled by a twice differentiable function of time, t. For $0 \le t \le 10$ min, the altitude h at various times is shown in the table.

t (min)	0	2	3	5	6	9	10
h (meters)	0	280	330	240	270	420	340

a) From the data shown, estimate the rate at which h is changing at $t = 7.5$. Show your work. Indicate units of measure.

b) Use a trapezoidal approximation with three subintervals to estimate the average height of the balloon during the first five minutes of its flight.

c) During the time interval $0 \le t \le 10$ min, what is the least number of times that $h'(t)$ must be zero? Justify your answer.

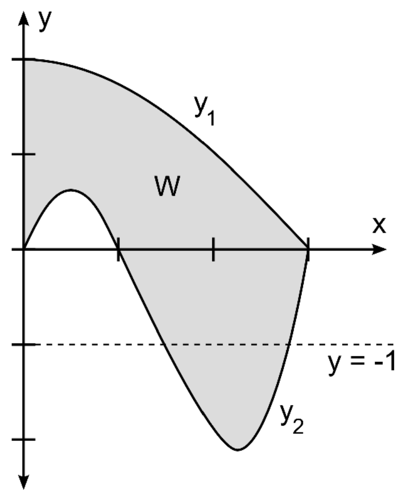
d) A pressurized propane tank supplies propane to the burner. The rate at which propane is dispensed is given by the function $8t \cdot e^{-0.4t}$ liters per minute. How many liters of propane are dispensed in the first five minutes?

2. The region W in the x-y plane is bound by

$$y_1 = 2\cos\left(\frac{\pi x}{6}\right) \text{ and}$$

$$y_2 = x^3 - 4x^2 + 3x \text{ and}$$

the y-axis, as shown in the diagram.



a) Find the area of region W.

b) The horizontal line $y = -1$ divides region W into two sections, Write an integral expression for the area of the lower section. Do not evaluate the integral.

c) Consider the region W to be the base of a solid S. The cross sections of the solid perpendicular to the x-axis are squares. Find the volume of S.

d) The region W is a top view of an experimental aircraft wing. The thickness of the wing at any distance x from the y-axis is given by the function $h(x) = 1 - \frac{x}{4}$. Find the volume of the wing.

End of Section 2, Part A

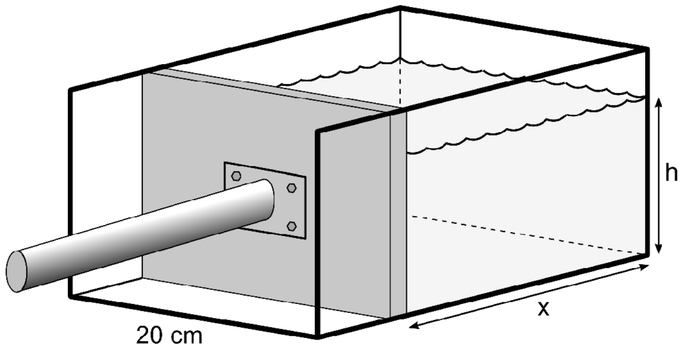
If you finish before the time limit for this part, check your work on this part only.

Do move on to the next part until you are told to by the test administrator.

Calculus AB
Section 2, Part B
Time - 60 minutes
Number of problems - 4

Calculator use is not permitted on these problems

3. A tank with a fixed width of 20 cm is built with a movable partition, as shown, which allows the length, x, of the tank to change. Water is being added to the tank at a constant rate of $120\ \frac{cm^3}{s}$. At the same time, the partition is moving. Both the length, x, and the height, h, of the water, are changing with time.

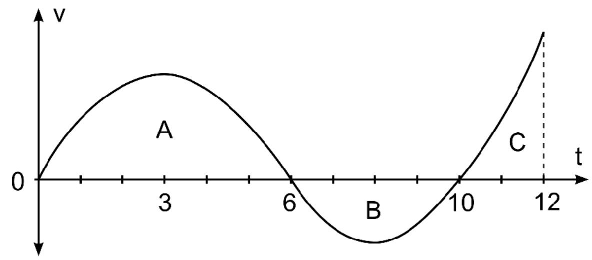


a) At the moment when $x = 40$ cm and $h = 10$ cm, x is increasing at $0.1\ \frac{\text{cm}}{\text{s}}$. At this moment, what is the rate of change of the height of the water in $\frac{\text{cm}}{\text{s}}$.

b) While water is being added to the tank at a constant rate of $120\ \frac{cm^3}{s}$, a device is introduced which begins to pump water out of the tank at $15\sqrt{t}\ \frac{\text{cm}^3}{\text{s}}$. At what time after the device starts working is the volume of water in the tank at a maximum. Justify your answer.

c) Suppose the pump starts operating when the volume of water is $16{,}000\ \text{cm}^3$. Write, but do not evaluate, an integral expression for the volume of water at the time when the volume is at a maximum.

4. An object moves along the x-axis. The graph of the object's velocity is the differentiable function shown below.



The regions A, B, and C, bounded by the graph and the t-axis, have areas of 10, 5, and 4 respectively. The graph has zeros at $t = 0$, $t = 6$, and $t = 10$, and has horizontal tangents at $t = 3$ and $t = 8$. At $t = 0$ the position of the object is $x = 5$.

a) At what point in time is x the greatest. Justify your answer.

b) For how many values of t is the object at position $x = 12$? Explain your reasoning.

c) During the time interval $9 < t < 10$, is the object's speed increasing or decreasing? Give a reason for your answer.

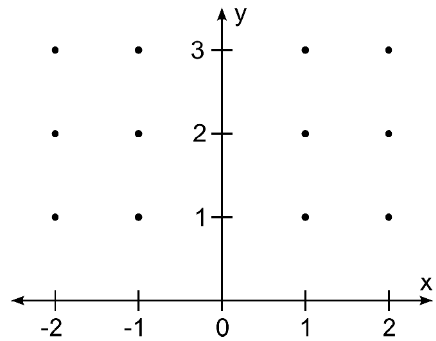
d) During what time intervals is the acceleration positive? Justify your answer.

5. For the differential equation $\frac{dy}{dx} = \frac{2 - y}{x^2}$

a) Sketch a slope field at the twelve points shown on the axes provided.

b) Find the particular solution to the equation with the initial condition $f(2) = 5$.

c) For the particular solution you found in part b, find $\lim_{x \to \infty} f(x)$.



6. Let $f(x)$ be the function defined as $f(x) = \dfrac{x}{\ln(x)}$ for all values of $x \geq 0$.

a) Find the derivative of $f(x)$.

b) Find an equation for the line tangent to $f(x)$ at $x = e^2$.

c) Function f has one relative extreme. Find the coordinates of this point. Determine whether it is a relative minimum or relative maximum.

d) Function f has one inflection point. Find the x-coordinate of this point.

e) Find $\lim_{x \to 0^+} f(x)$.

End of Exam

AP Calculus AB

Practice Exam
Answers

Answers to Multiple Choice Questions

Section 1

Part A

1.	A
2.	C
3.	B
4.	C
5.	A
6.	D
7.	B
8.	E
9.	D
10.	A
11.	D
12.	C
13.	E
14.	C
15.	B
16.	C
17.	D
18.	A
19.	C
20.	C
21.	B
22.	D
23.	E
24.	D
25.	A
26.	A
27.	C
28.	D

Part B

76.	B
77.	A
78.	D
79.	A
80.	D
81.	C
82.	A
83.	C
84.	C
85.	E
86.	E
87.	B
88.	C
89.	B
90.	E
91.	C
92.	D

SOLUTIONS

Calculus AB
Section 2, Part A
Time - 30 minutes
Number of problems - 2

A graphing calculator is required for these problems

1. A hot air balloon is launched at time $t = 0$. Its altitude in meters is modeled by a twice differentiable function of time, t. For $0 \le t \le 10$ min, the altitude h at various times is shown in the table.

t (min)	0	2	3	5	6	9	10
h (meters)	0	280	330	240	270	420	340

a) From the data shown, estimate the rate at which h is changing at $t = 7.5$. Show your work. Indicate units of measure.

b) Use a trapezoidal approximation with three subintervals to estimate the average height of the balloon during the first five minutes of its flight.

c) During the time interval $0 \le t \le 10$ min, what is the least number of times that $h'(t)$ must be zero? Justify your answer.

d) A pressurized propane tank supplies propane to the burner. The rate at which propane is dispensed is given by the function $8t \cdot e^{-0.4t}$ liters per minute. How many liters of propane are dispensed in the first five minutes?

---

a) To find the rate of change of h at $t = 7.5$, use the values just before and after 7.5.

$$\frac{\Delta h}{\Delta t} = \frac{420 - 270}{9 - 6} = \frac{150}{3} = 50 \ \frac{\text{m}}{\text{min}}$$

b) Use a trapezoid sum, *not* the Trapezoid Rule.

$$\int_0^5 h(t) = \tfrac{1}{2}(0 + 280) \cdot 2 + \tfrac{1}{2}(280 + 330) \cdot 1 + \tfrac{1}{2}(330 + 240) \cdot 2$$

$$= 280 + 305 + 570 = 1155$$

$$\text{average height} = \frac{1}{5 - 0} \cdot \int_0^5 h(t)\,dt = \frac{1155}{5} = 231 \text{ meters}$$

c) The altitude increases from 0 to 3 seconds, then decreases from 3 to 5, then increases from 5 to 9, then decreases from 9 to 10. In other words, the balloon goes up, down, up, down. The rate of change of h must be zero at least three times.

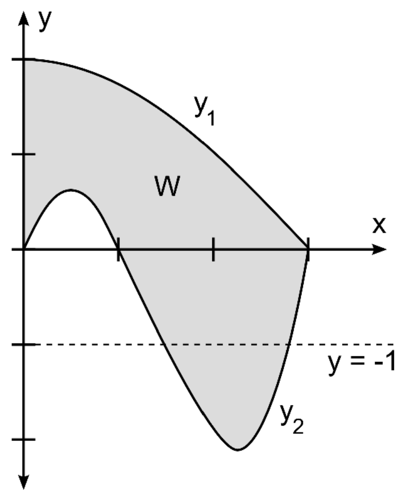
d) On the calculator: $\int_0^5 \left(8t \cdot e^{-0.4t}\right) dt = 29.7$ liters

2. The region W in the x-y plane is bound by

$$y_1 = 2\cos\left(\frac{\pi x}{6}\right) \text{ and}$$

$$y_2 = x^3 - 4x^2 + 3x \text{ and}$$

the y-axis, as shown in the diagram.



a) Find the area of region W.

b) The horizontal line $y = -1$ divides region W into two sections, Write an integral expression for the area of the lower section. Do not evaluate the integral.

c) Consider the region W to be the base of a solid S. The cross sections of the solid perpendicular to the x-axis are squares. Find the volume of S.

d) The region W is a top view of an experimental aircraft wing. The thickness of the wing at any distance x from the y-axis is given by the function $h(x) = 1 - \frac{x}{4}$. Find the volume of the wing.

---

a) The integral is evaluated fairly quickly on the calculator:

$$A = \int_0^3 (y_1 - y_2)\,dx = 6.07$$

b) The line $y = -1$ intersects y_2 at two points. To find these points, graph $y_2 + 1$ on the calculator and find the zeros: $x = 1.445$ and $x = 2.802$.

$$A = \int_{1.445}^{2.802} \left(-1 - y_2\right)dx \qquad \text{or} \qquad A = -\int_{1.445}^{2.802} \left(x^3 - 4x^2 + 3x + 1\right)dx$$

c) $dV = \left(y_1 - y_2\right)^2 dx$

The integral is evaluated on the calculator: $V = \int_0^3 \left(y_1 - y_2\right)^2 dx = 13.739$

d) $dV = \left(y_1 - y_2\right)\left(1 - \frac{x}{4}\right)dx$

$$\int_0^3 \left(y_1 - y_2\right)\left(1 - \frac{x}{4}\right)dx = 3.679$$

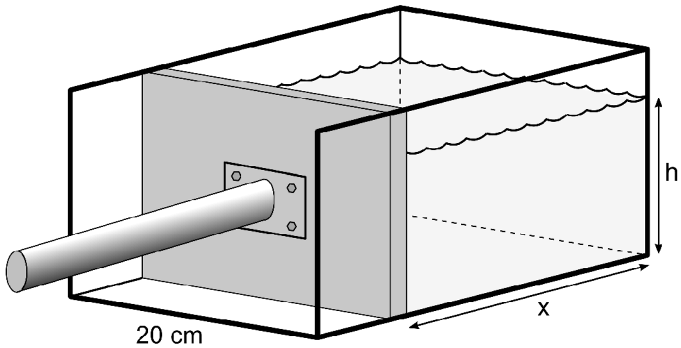
End of Section 2, Part A

SOLUTIONS

Calculus AB
Section 2, Part B
Time - 60 minutes
Number of problems - 4

Calculator use is not permitted on these problems

3. A tank with a fixed width of 20 cm is built with a movable partition, as shown, which allows the length, x, of the tank to change. Water is being added to the tank at a constant rate of $120\ \frac{cm^3}{s}$. At the same time, the partition is moving. Both the length, x, and the height, h, of the water, are changing with time.



a) At the moment when $x = 40$ cm and $h = 10$ cm, x is increasing at $0.1\ \frac{\text{cm}}{\text{s}}$. At this moment, what is the rate of change of the height of the water in $\frac{\text{cm}}{\text{s}}$.

b) While water is being added to the tank at a constant rate of $120\ \frac{cm^3}{s}$, a device is introduced which begins to pump water out of the tank at $15\sqrt{t}\ \frac{\text{cm}^3}{\text{s}}$. At what time after the device starts working is the volume of water in the tank at a maximum. Justify your answer.

c) Suppose the pump starts operating when the volume of water is $16{,}000\ \text{cm}^3$. Write, but do not evaluate, an integral expression for the volume of water at the time when the volume is at a maximum.

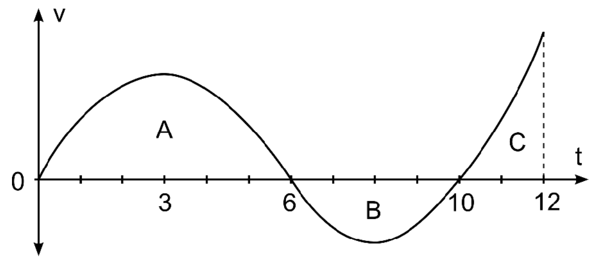
---

a)
$$V = 20xh$$
$$\dot{V} = 20(x\dot{h} + h\dot{x})$$
$$= 20x\dot{h} + 20h\dot{x}$$
$$\dot{h} = \frac{\dot{V} - 20h\dot{x}}{20x} = \frac{120 - 20(10)(0.1)}{20(40)} = \frac{100}{800} = \frac{1}{8}$$

b) V will be increasing until the rate at which the water is being pumped exceeds 120.
$$15\sqrt{t} = 120$$
$$\sqrt{t} = 8$$
$$t = 64 \text{ sec}$$

c)
$$V = 16{,}000 + \int_0^{64}\left(120 - 15\sqrt{t}\right)dt$$

4. An object moves along the x-axis. The graph of the object's velocity is the differentiable function shown below.



The regions A, B, and C, bounded by the graph and the t-axis, have areas of 10, 5, and 4 respectively. The graph has zeros at $t = 0$, $t = 6$, and $t = 10$, and has horizontal tangents at $t = 3$ and $t = 8$. At $t = 0$ the position of the object is $x = 5$.

a) At what point in time is x the greatest. Justify your answer.

b) For how many values of t is the object at position $x = 12$? Explain your reasoning.

c) During the time interval $9 < t < 10$, is the object's speed increasing or decreasing? Give a reason for your answer.

d) During what time intervals is the acceleration positive? Justify your answer.

---

a) From 0 to 6, the object moves forward 10.
From 6 to 10s, it moves backward 5.
From 10 to 12s, it moves forward 4.
x will be greatest at $t = 6$.

b) Based on the distances, the object starts at $x = 5$, moves forward to 15, backward to 10, then forward to 14. It will cross the position $x = 12$ three times.

c) From 9 to 10, the absolute value of v decreases, so the speed is decreasing. The correct answer to this question involves the distinction between speed and velocity. Even though the velocity is increasing the speed is decreasing.

d) The acceleration is positive whenever the v graph is rising. This occurs when

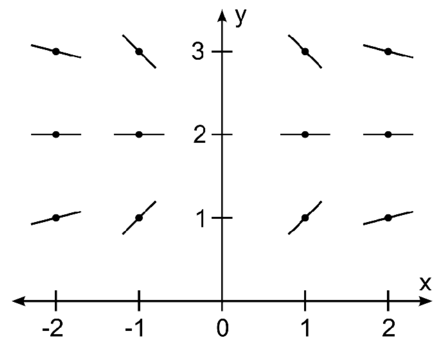
$$0 < t < 3 \text{ and } 8 < t < 12.$$

5. For the differential equation $\dfrac{dy}{dx} = \dfrac{2 - y}{x^2}$

a) Sketch a slope field at the twelve points shown on the axes provided.

b) Find the particular solution to the equation with the initial condition $f(2) = 5$.

c) For the particular solution you found in part b, find $\lim_{x \to \infty} f(x)$.



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a) See slope field plotted above

b) $$\int \frac{dy}{2 - y} = \int x^{-2}\, dx$$

$$\ln(2 - y) = -\frac{1}{x} + C$$

$$2 - y = e^{-1/x + C} = e^{-1/x} e^{C} = ke^{-1/x}$$

$$y = 2 - ke^{-1/x}$$

Using the initial condition $x = 2$, $y = 5$:

$$5 = 2 - ke^{-1/2}$$

$$k = \frac{3}{e^{-1/2}} = 3e^{1/2}$$

$$y = 2 - 3e^{1/2} e^{-1/x}$$

$$y = 2 - 3e^{1/2 - 1/x}$$

c) $$\lim_{x \to \infty} y = 2 - 3e^{1/2} = 2 - 3\sqrt{e}$$

6. Left $f(x)$ be function defined as $f(x) = \dfrac{x}{\ln(x)}$ for all values of $x \geq 0$.

   a) Find the derivative of $f(x)$.

   b) Find an equation for the line tangent to $f(x)$ at $x = e^2$.

   c) Function f has one relative extreme. Find the coordinates of this point. Determine whether it is a relative minimum or relative maximum.

   d) Function f has one inflection point. Find the x-coordinate of this point.

   e) Find $\lim_{x \to 0^+} f(x)$.

---

a) $$f'(x) = \frac{\ln(x) \cdot 1 - x \cdot \frac{1}{x}}{\ln^2(x)} = \frac{\ln(x) - 1}{\ln^2(x)}$$

b) $$f'(e^2) = \frac{e^2}{\ln(e^2)} = \frac{e^2}{2}$$

$$f'(e^2) = \frac{\ln(e^2) - 1}{\ln^2(e^2)} = \frac{2\ln e - 1}{(2\ln e)^2} = \frac{2 - 1}{4} = \frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{e^2}{2} = \frac{1}{4}(x - e^2)$$

or $$y = \frac{1}{4}x + \frac{e^2}{4}$$

c) The relative extreme occurs when $f'(x) = 0$. This happen when $\ln x = 1$.

$f(e) = \dfrac{e}{\ln e} = e$, so the point is $(e,\ e)$. The coordinates can also be found on the graphing calculator, where it is apparent that the point is a relative minimum.

d) $$f''(x) = \frac{\ln^2 x \cdot \frac{1}{x} - (\ln x - 1) \cdot 2\ln x \cdot \frac{1}{x}}{\ln^4 x}$$

This equals zero when

$$\frac{\ln x}{x}\left(\ln x - 2(\ln x - 1)\right) = 0$$

$$-\ln x + 2 = 0$$

$$\ln x = 2$$

$$x = e^2$$

Graphing the second derivative on a graphing calculator and finding the zero numerically results in an x value of 7.389, which is equivalent to e^2, but the above analysis may be faster.

e) As x approaches zero from the right, the numerator of f gets close to zero while the denominator approaches negative infinity. Therefore

$$\lim_{x \to 0^+} f(x) = 0$$

End of Exam